SR No - 2764/2022

Name of the Course : B. A. (Prog.)
Unique Paper Code : 62357603
Name of the Paper : Numerical Methods
Semester : VI
Duration : 3 Hours
Maximum Marks : 75 Marks
Duration: 3 hours
(Write your roll no. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question.
Use of Non – Programmable Scientific Calculator is allowed.

Q-1. (a) Find the root of the equation $f(x) = \cos x - xe^x$ lying between (0,1) by the Bisection method. [6]

(b) Obtain the rate of Convergence of Regula - Falsi method. [6]

(c) Suppose 1.414 is used as an approximation to $\sqrt{2}$, then find the absolute and relative error. [6]

(d) Find a real root of the equation given by $x^2 - 7x + 1 = 0$ by Newton – Raphson Method. [6]

Q-2. (a) Solve the given equation $f(x) = x^3 + x^2 + x + 6$ by Secant method. [6.5]

(b) Perform two iterations of Newton's method to solve the non – linear system of equation with initial approximation (1,1) :-

$$f(x,y) = x^2 + y^2 - 4 = 0,$$
$$g(x,y) = x^2 + y^2 - 16 = 0$$ [6.5]

(c) Round off the number 784320 to four significant digits and compute Ea (Absolute error), Er (Relative error) and Ep (Percentage error) for this number. [6.5]

(d) Find the real root of the equation $f(x) = x^3 - 9x + 2 = 0$ by Regula – Falsi method in the interval 2 and 3. [6.5]

Q-3. (a) Starting with initial vector $(x, y, z) = (0, 0, 0)$, perform three iterations of Gauss-Seidel method to solve the following system of equation:

$$2x - y = 7$$
$$-x + 2y - z = 1$$
$$-y + 2z = 1.$$ [6]

(b) Construct the interpolating polynomial using the Gregory-Newton backward difference interpolation for the given data set: [6]

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.4	1.5	1.7	2.0	2.2

Hence estimate the value of $f(x)$ at $x = 0.35$.

(c) Using Gauss-Jordan method, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$ [6]

(d) Consider the following table: [6]

x	1	2	4	5
$f(x)$	5.3	2.0	3.1	1.0

Use dividend difference to calculate the interpolating polynomial of degree 3 and give an estimate for $f(1.5)$.

Q-4. (a) Find the Lagrange interpolation polynomial for the data set: (0, 1), (1, 3) and (3, 5.5). Also estimate the value at $x = 1.5$. [6.5]

(b) Using Gaussian Elimination method, solve the following system of equations:

$$4x - 3y + z = -8$$
$$-2x + y - 3z = -4$$
$$x - y + 2z = 3.$$

[6.5]

(c) For the following system of equations:

$$4x + y + z = 2$$
$$x + 5y + 2z = -6$$
$$x + 2y + 3z = -4.$$

Use Gauss-Jacobi iteration method by performing three iterations. Take the initial approximation as $(x, y, z) = (1,1,1)$. [6.5]

(d) Prove the following relation:

$$\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}.$$ [6.5]

Also construct the forward difference table at the following data points:

x	0	1	2	3	4
$f(x)$	1	7	23	55	109

Q-5. (a) The velocities of a car running on a straight road at intervals of 2 minutes are given below:

Time(min)	0	2	4	6	8	10	12
Velocity(km/hr)	0	22	30	27	18	7	0

Using Simpson $\frac{1}{3}$rd rule find the distance covered by the car. [6]

(b) Given that $\frac{dy}{dx} - \sqrt{xy} = 2$ with $y(1) = 1$, find $y(2)$ by Euler's method (Take $h = 0.5$). [6]

(c) Using Midpoint method obtain the numerical solution $y(0.2)$ of the initial value problem $y' = (x - y)^2; y(0) = 0.5$ correct to 4 decimal places, with $h = 0.1$. [6]

(d) Calculate the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 10$; given the following table:- [6]

x	3	5	11	27
f	-13	23	89	170

Q-6. (a) Evaluate $\int_0^1 \frac{1+x}{1+x^3} dx$ using Trapezoidal rule by dividing interval into five equal parts. [6.5]

(b) Given that the differential equation $\frac{dy}{dx} = x^2 + y, y(0) = 1$. Find the value of $y(0.02)$, using Euler's modified method. [6.5]

(c) Find $f'(x_2)$ using central difference formula and Richardson extrapolation with $h = 2, (x_0, y_0) = (1,2), (x_1, y_1) = (2,4), (x_2, y_2) = (3,8), (x_3, y_3) = (4,16), (x_4, y_4) = (5,32)$. [6.5]

(d) Find the value of the integral $I = \int_0^4 \frac{dx}{1+x^2}$ using the Simpson 3/8th rule with $h = 1$. [6.5]